1. Two infinite parallel plates, separated by a distance s, are at $\Phi = 0$ and Φ_0 respectively.
(a) Use Poisson's equation to find the potential in the region between the plates where the space charge density is $\rho(x) = \rho_0 x/s$. The distance x is measured from the plate at zero potential.
(b) Find the surface charge densities on the plates.

2. (a) Two coaxial conducting cylinders, of radii a and b ($b > a$) and length L ($L \gg b$), carry equal and opposite charges $-Q$ and Q. Find the capacitance of this configuration.
(b) Find the electrostatic energy of the configuration by integrating over the distribution. Express your answer in terms of Q and $\Delta\Phi$.

3. Consider the electrostatic Green's function on the surface bounding the volume V. Using the Green's theorem with integration variables $\vec{y}$ and $\varphi = G(\vec{\boldsymbol{x}}', \vec{\boldsymbol{y}})$, with $\vec{\boldsymbol{\nabla}}_y^2 G(\vec{\boldsymbol{z}}, \vec{\boldsymbol{y}}) = -4\pi\delta(\vec{y} - \vec{z})$, find an expression for the difference $[G(\vec{\boldsymbol{x}}, \vec{\boldsymbol{x}}') - G(\vec{\boldsymbol{x}}', \vec{\boldsymbol{x}})]$ in terms of an integral over the surface S.

4. A point charge q is brought to a position a distance d away from an infinite plane conductor held at zero potential. Using the method of images,
(a) verify that the total charge induced on the plane is equal to $-q$;
(b) determine the total force acting on the plane by integrating $\sigma^2/2\epsilon_0$;
(c) determine the work necessary to remove the charge q from its position to infinity using $\int \vec{\boldsymbol{F}} \bullet d\vec{\boldsymbol{\ell}}$;
(d) determine the potential energy between the charge q and its image if there was no conducting plane present. Compare the results with part (c) and explain your result.

5. (a) Show that the electrostatic potential at $\vec{\boldsymbol{x}}$ due to a dipole $\vec{\boldsymbol{p}}$ at the origin is

$$\Phi(\vec{\boldsymbol{x}}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{\boldsymbol{p}} \bullet \vec{\boldsymbol{x}}}{|\vec{\boldsymbol{x}}|^3}$$

(b) Using $\vec{\boldsymbol{E}} = -\vec{\boldsymbol{\nabla}}\Phi$, show that

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\epsilon_0} \frac{3\vec{\boldsymbol{x}}(\vec{\boldsymbol{x}} \bullet \vec{\boldsymbol{p}}) - \vec{\boldsymbol{p}} r^2}{r^5}$$

(c) If $\vec{\boldsymbol{p}} = p\,\hat{\boldsymbol{k}}$, find E_r, E_θ, and E_ϕ.